Split Plot Design.

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Introduction

- In some multifactor factorial experiments we may be unable to completely randomize the order of runs.
- This often results in a generalization of the factorial design called a **split-plot design**
- The linear model for the split-plot design is

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \gamma_k + (\tau\gamma)_{ik} + (\beta\gamma)_{jk} + (\tau\beta\gamma)_{ijk} + \epsilon_{ijk}$$

$$i = 1, 2 ... r, j = 1, 2, ... a, k = 1, 2, ... b$$

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- where τ_i, β_j and $(\tau, \beta)_{ij}$ represent the whole plot and correspond respectively to replicates, main treatments (factor A) and whole plot error (replicates $\times A$) and $\gamma_k, (\tau\gamma)_{ik}, (\beta\gamma)_{jk}$ and $(\tau\beta\gamma)_{ijk}$ represents the subplot and correspond respectively to the subplot treatment (factor B), the replicates $\times B$ and AB interactions and the subplot error (replicates $\times$ AB)

Advantages of a split plot design in experimental design.

- **Cost**: The cost of running a set of treatments in splitplot order is generally less than the cost of the same experiment when completely randomized.
- **Efficiency**: Split-plot experiments are not just less expensive to run than completely randomized experiments; they are often more efficient statistically.
- **Validity**: Completely randomized designs are prescribed frequently in industry, but are typically not run as such in the presence of hard-to-change factors

Analysis

Whole-Plot Analysis

- This part of the analysis is based on comparison of whole-plot totals.
- The levels of A are assigned to whole plots within blocks according to randomized complete block design and so the sum of squares for A needs no block adjustment.
- There are $a - 1$ degrees of freedom for A so the sum of squares are given by:

$$SS_A = \frac{\sum_j y_{.j.}^2}{rb} - \frac{y_{...}^2}{rab}$$

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- There are $r-1$ degrees of freedom for the blocks giving block sum of squares

$$SS_R = \frac{\sum_i y_i^2}{ab} - \frac{y_{...}^2}{rab}$$

- There are a whole plots nested within each of the r blocks so there are in total $r(a-1)$ whole-plot degrees of freedom.
- Of these $a-1$ are used to measure the effects of A leaving $(r-1)(a-1)$ degrees of freedom for whole-plot error.

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- Equivalently this can be obtained by the subtraction of the block and A degrees of freedom from the whole-plot total degrees of freedom i.e $(ra-1)-(r-1)-(a-1)=(r-1)(a-1)$
- So the whole plot error sum of squares is obtained:

$$SS_{E1}=\frac{\sum_i\sum_j y_{ij}^2}{b}-\frac{y_{...}^2}{rab}-SS_R-SS_A$$

- The whole plot error mean square

$$MSE_1=\frac{SSE_1}{(r-1)(a-1)}$$

is used as the error estimate to test the significance of whole plot factor A

Sub-Plot Analysis

- This part of the analysis is based on the observations arising from the split-plots within whole-plots
- There are $rab - 1$ total degrees of freedom and the total sum of squares is

$$SST = \sum_i \sum_j \sum_k y_{ijk}^2 - \frac{y_{...}^2}{rab}$$

- Sum of squares for B is given by:

$$SS_B = \frac{\sum_k y_{..k}^2}{ra} - \frac{y_{...}^2}{rab}$$

corresponding to $b - 1$ degrees of freedom

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- The interaction degrees of freedom is given by

$$SS_{AB} = \frac{\sum_i \sum_j y_{.jk}^2}{r} - \frac{y_{...}^2}{rab} - SS_A - SS_B$$

- The split-plot error sum of squares can be calculated as

$$SSE_2 = SST - SS_R - SS_A - SSE_1 - SS_B - SS_{AB}$$

with $a(r-1)(b-)$

ANOVA Table

Source	Df	SS	MSS	F
Whole Plot Analysis				
Replication	r-1	SSR		
Main Plot Treatment (A)	a-1	SSA	MSA	$\frac{MSA}{MSE_1}$
Main plot error E_1	(r-1)(a-1)	SSE_1	MSE_1	
Sub-plot Analysis				
Sub-plot treatment(B)	b-1	SSB	MSB	$\frac{MSB}{MSE_2}$
Interaction	(a-1)(b-1)	SSAB	MSAB	$\frac{MSAB}{MSE_2}$
Sub-plot error E_2	a(r-1)(b-1)	SSE_2	MSE_2	
Total	rab-1	SST		

Example

In a study carried out by agronomists to determine if major differences in yield response to N fertilization exist among different varieties of jowar the main plot treatments were three varieties of jowar V1, V2 and V3 and the sub-plot treatments were N rates of 0, 30 and 60 Kg/ha. The study was replicated four times and the data shown in Table 1

Data

		N rate, Kg/ha		
Replication	Variety	0	30	60
I	V1	15.5	17.5	20.8
	V2	20.5	24.5	30.2
	V3	15.6	18.2	18.5
II	V1	18.9	20.2	24.5
	V2	15.0	20.5	18.9
	V3	16.0	15.8	18.3
III	V1	12.9	14.5	13.5
	V2	20.2	18.5	25.4
	V3	15.9	20.5	22.5
IV	V1	12.9	13.5	18.5
	V2	13.5	17.5	14.9
	V3	12.5	11.9	10.5

Steps of Analysis

Calculate the replication totals (R) and the grand total (G) by first constructing a table for the replication $\times$ variety totals

	Variety			
Replication	V1	V2	V3	Replication Total(R)
I	53.8	75.2	52.3	181.3
II	63.6	54.4	50.1	168.1
III	40.9	64.1	58.9	163.9
IV	44.9	45.9	34.9	125.7
Variety Total (A)	203.2	239.6	196.2	
Grand Total G				639.0

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We construct a second table for the variety $\times$ nitrogen totals

	Variety			
	V1	V2	V3	Nitrogen Total (B)
N_0	60.2	69.2	60.0	189.4
N_1	65.7	81.0	66.4	213.1
N_2	77.3	89.4	69.8	236.5

Various Sum of Squares

- The correction factor

$$C.F = \frac{G^2}{rab} = \frac{639 \times 639}{4 \times 3 \times 3} = 11342.25$$

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$$SST = [(15.5)^2 + (20.5)^2 + ... + (10.5)^2] - CF = 637.97$$

$$Replication \;\; Sum \;\; of \;\; Squares =$$

$$\frac{\sum R^2}{ab} - C.F = \frac{(181.3)^2 + (168.1)^2 + (163.9)^2 + (125.7)^2}{3 \times 3} - 11342.25$$

$$= 190.08$$

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Sum of Squares due to variety (SSA)

$$SSA = \frac{\sum A^2}{rb} - CF = \frac{(203.2)^2 + (239.6)^2 + (196.2)^2}{4 \times 3} - 11324.25 = 90.487$$

Main Plot error

$$SSE_1 = \frac{\sum (RA)^2}{b} - C.F - SSR - SSA$$

$$\frac{(53.8)^2 + (63.6)^2 + ... + (34.9)^2}{3} - 11342.25 - 190.08 - 90.487 = 174.103$$

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Sum of squares due to Nitrogen (SSB)

$$SSB = \frac{\sum B^2}{ra} - C.F = \frac{(189.4)^2 + (213.1)^2 + (236.5)^2}{4 \times 3} - 11342.25 = 92.435$$

Sum of Squares due to interaction

$$SS_{AB} = \frac{\sum (AB)^2}{r} - CF - SSA - SSB$$

$$= \frac{(60.2)^2 + (65.7)^2 + ... + (69.8)^2}{4} - 11342.25 - 90.487 - 92.435 = 9.533$$

Sub plot error $SSE_2 = SST - All \ \ other \ \ sum \ \ of \ \ squares$

$$SSE_2 = 637.97 - 190.08 - 90.487 - 174.103 - 92.435 - 9.533 = 81.332$$

ANOVA Table

Source	DF	SS	MSS	F
Replication	3	190.08	63.360	
Variety (A)	2	90.487	45.243	1.56
Error(a)	6	174.103	29.017	
Nitrogen (B)	2	92.435	46.218	10.23
$Variety \times Nitrogen$	4	9.533	2.383	
Error(b)	18	81.332	4.518	
Total	35	637.97		

Thank You!